

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2025 (As Per Syllabus Year June-2023)
MAT: Matrices and Coordinate Geometry (Minor)

Time: 2:00 Hours

Marks:50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one (05)

- (1) Define: Trace of a matrix, triangular matrix, Skew-Hermitian Matrix, Orthogonal Matrix, Nilpotent Matrix.
- (2) In usual notations show that $(AB)^T = B^T A^T$; Where A and B are matrices of the same order.

Que.1(B) Answer any one (05)

- (1) Prove that every square matrix can be expressed Uniquely as the sum of a symmetric matrix and skew Symmetric matrix.
- (2) If $AB=BA$ then prove that $(AB)^n = A^n B^n$; $n \in \mathbb{N}$.

Que.2(A) Answer any one (05)

- (1) Prove that rank of any Matrix $\geq$ rank of its every submatrix.
- (2) Prove that the rank of the product of two matrices cannot exceed the rank of either matrix.

Que.2(B) Answer any one (05)

- (1) Find inverse of Matrix $\begin{bmatrix} 5 & 8 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary row operations.

- (2) Find Rank of Matrix $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$

Que.3(A) Answer any one (05)

- (1) State and prove Cayley- Hamilton theorem.
- (2) Prove that there is a unique eigen value with respect to any eigen vector of given matrix.

Que.3(B) Answer any one (05)

- (1) Prove that the equation $AX=0$ has a non-zero solution iff A is singular.
- (2) Solve equations if possible:

$$x + y + z = -3$$

$$3x + y - 2z = -2$$

$$2x + 4y + 7z = 7$$

Que.4(A) Answer any one (05)

- (1) Derive the formula to find distance between two points in polar co-ordinates.
- (2) Show that $(0,0), (3, \frac{\pi}{2})$ and $(3, \frac{\pi}{6})$ are vertices of an equilateral triangle.

Que.4(B) Answer any one (05)

- (1) Obtain the equation of the circle whose center is (ρ, α) and radius a .
- (2) Find the spherical coordinates for the point $(-\sqrt{3}, -1, 2\sqrt{3})$

Que.5(A) Answer any one (05)

- (1) Find echelon form of matrix $\begin{bmatrix} 1 & 0 & -4 \\ 0 & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$.
- (2) Find inverse of matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ using Cayley – Hamilton theorem.

Que.5(B) Answer any one (05)

- (1) Find Eigen values and Eigen vectors of matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
- (2) Derive relation between Cartesian Coordinates and Cylindrical Coordinates.
